SIMPLE METHOD TO ESTIMATE REPEATER SPAN BY VARYING THE LENGTH OF A REFERENCE DIGITAL LINE SECTION.

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Abstract: In this paper, we review a simple heuristic method to estimate the required repeater span of a new cable system from the knowledge of the span, overall length and fibre attenuation of a system already deployed in the field, and we analyse its range of validity by comparing it to an accurate simulation model. We show that by taking appropriate care, the estimation error can be kept low enough to be used for the investment opportunity studies needed to drive new cable system deployments.

1. INTRODUCTION

When studying an opportunity to invest in a new cable system, submarine cable operators must conduct techno-economic opportunity studies to explore different architecture options and define a cost-effective project. The key point for such studies is to be able to estimate the cost of the Digital Line Sections (DLS) of a projected cable system, one of the key parameters being the number of repeaters.

When deploying a totally new technology, purchasers have to rely on the line design studies of their potential suppliers or on their internal R&D technical teams. It requires a lot of technical information related to the transponders and fibre characteristics [1] that can be difficult to collect when the technology is not yet deployed.
But with technology already deployed in the field, if they have access to technical and price information of one or more existing cable systems, they can use a simple heuristic method to estimate the repeater span from an existing reference DLS for which the repeater span is known. This approach ignores a number of technical parameters and avoids the use of complex simulation tools. The price to pay for such simplicity is a loss of accuracy in the repeater number estimation, therefore the approach can only be used as a first estimation step and in no way as a line design tool.

In this paper, we review the heuristic method which is based on a Signal to Noise Ratio (SNR) conservation principle, in the context of coherent technology and non-dispersion managed fibre map. We analyse its range of validity by comparing it to an accurate simulation model.

2. SIMPLE APPROXIMATION OF SPAN AND DLS LENGTH

Let respectively s_0, n_0, l_0 and s, n, l denote the span length, the number of repeaters and the overall length, respectively for a reference DLS and for a target DLS, and let α denote the fibre attenuation in dB/km. A simple way to have an estimation of a value of s and n to reach the target system length l is to equalize the amplifier noise accumulation between both systems. We can easily approximate such amplified spontaneous emission (ASE) noise accumulation by $P_{ASE} = \eta_{ASE}\,.\,n\,.\,G$ where $\eta_{ASE} = hfN_FB_0$ with h, f, N_F and B_0 being respectively the Planck constant, the channel frequency, the mean amplifier noise figure and the noise reference bandwidth. G denotes the amplifiers gain which is also equal to the span loss on the dB scale: $G_{dB} = \alpha\, s$. Equating P_{ASE} for both systems yields:

$$n_0\,.\,G_0 = n\,.\,G$$

If we consider the repeater output power is constant, the above equation is equivalent to expressing that the SNR of the reference system and the target system are equal. Translating the equation to the dB scale we obtain equation (1) expressing that the noise power must stay constant, from which we derive formula (2) giving the span length:

$$\alpha\,(s - s_0) + 10\,\log_{10}\frac{n}{n_0} = 0 \qquad (1)$$

$$s = s_0 - \frac{1}{\alpha}\,10\,\log_{10}\frac{n}{n_0} \qquad (2)$$

The following strong assumptions have been made:

- the nonlinear penalty doesn’t scale with n and s,
- the optical power per channel is the same for both systems.

If we express the DLS length as a function of the number of repeaters, we get the formula:

$$\begin{aligned} l &= n\,.\left(s - s_0 + \frac{l_0}{n_0}\right) \\ &= n\,.\left(\frac{l_0}{n_0} - \frac{1}{\alpha}\,10\,\log_{10}\frac{n}{n_0}\right) \qquad (2') \end{aligned}$$

In fact, the system length cannot be directly calculated as $l = n\,.\,s$ because a few spans may include gain equalizers with intrinsic attenuation, so they have shorter lengths. In equation (2’), we make the assumption that the new system has the same period of equalization than the reference one.

3. APPROXIMATION OF SPAN AND DLS LENGTH TAKING INTO ACCOUNT BOTH LINEAR AND NON LINEAR NOISE

In fact, the system has to be optimized with respect to two noise sources, the additive ASE noise power P_{ASE} brought by the EDFA amplifiers in the repeaters, and the non linear noise power P_{NL} due to the non linear interferences occurring during the light propagation in the fibre [2]. The performance of the system can therefore be characterized by a “total” signal to noise ratio:

$$SNR = \frac{P}{P_{ASE}+P_{NL}} \qquad (3)$$

There is an optimal value of the output power P maximizing the SNR, below which the repeater noise P_{ASE} is the limiting factor, and above which the power noise P_{NL} is the limiting factor.

A reasonable assumption is that the reference DLS has been designed by the wet plant supplier to operate with an optimal power P_0^{opt} and that we are also looking for a new DLS operating at a new optimal power P^{opt}.

To determine the P^{opt} minimizing the number of repeaters for a given DSL length, we have conducted extensive simulations using the Gaussian Noise (GN) model proposed in [3] with the system parameters summarized in Table 1. The systems performances are simulated with 80 channels, 50 GHz spaced and carrying 128 Gbit/s using a Quaternary Phase Shift Keying (QPSK). The back to back characteristics of channels have been experimentally measured and integrated in the simulation tool.

Fiber Parameters	
Effective Area (A_{eff})	112 µm²
Attenuation (α)	0,162 dB/km
Chromatic dispersion (D)	20,4 ps/(nm.km)
Modulation Parameters	
bit rate	128 Gbit/s
Δλ	50 GHz

Table 1: Simulation parameters

FEC limit	5,5 dB
Manufacturing impairment	0,5 dB
Gain, wavelength and supervisory imp.	0,5 dB
Time System Varying Performances	0,5 dB
Aging and repairs	1 dB
EOL customer margin	1 dB
Design limit	9 dB

Table 2: Design Q factor limit considered in the simulation

For each span length, we have determined the optimal optical power for the maximum number of span the system can reach while keeping a Q factor higher than the design limit we have estimated to 9 dB following the assumption given in the Table 2. From the Figure 1, we have observed that on the dB scale P^{opt} increases in a quasi-linear way with respect to the span length s with a slope β = 0.06 dB/km.

Considering that we have $P_{ASE} = 2P_{NL}$ at optimal power [2], we have the following expression:

$$SNR = \frac{P}{\frac{3}{2}\eta_{ASE} G\, n}$$

We obtain after equating the SNR of the target and reference systems the relation:

$$\frac{P_0}{P} = \frac{G_0\, n_0}{G\ \ n}$$

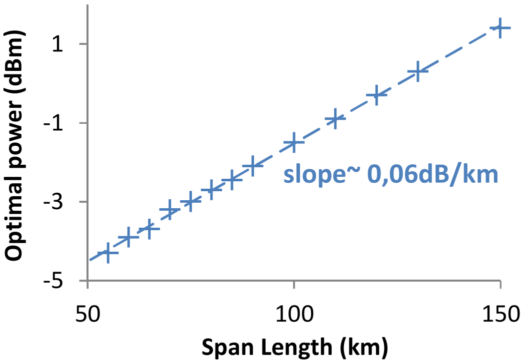

Figure 1: Optimal channel power simulated for different span lengths

Therefore equation (1) must replaced by a new equation also involving coefficient β and expressing that the noise power varies by the same amount as the repeater output power:

$$\alpha(s - s_0) + 10\ \log_{10}\frac{n}{n_0} = \beta(s - s_0)\ (4)$$

We obtain the new expressions of s and l with the attenuation α replaced by a corrected value $\alpha - \beta$:

$$s = s_0 - \frac{1}{\alpha-\beta}\ 10\ \log_{10}\frac{n}{n_0} \qquad (5)$$

$$l = n\,.\left(\frac{l_0}{n_0} - \frac{1}{\alpha-\beta}\ 10\ \log_{10}\frac{n}{n_0}\right) \qquad (5')$$

4. RFERENCE SYSTEM

A reference system has to be determined to compare the results of formula (5) and the simulation model. For confidentiality reasons with respect to existing systems deployed recently, we have built a reference system using a lab experiment. A recirculating loop of 10 spans of 76 km of G654.D has been built with the same characteristics of the simulation parameters given in Table 1. 2x26 channels with 100 GHz spacing have been independently modulated with 128 Gbit/s QPSK thanks to two lab transmitters. Odd and even channels have then been interleaved with a

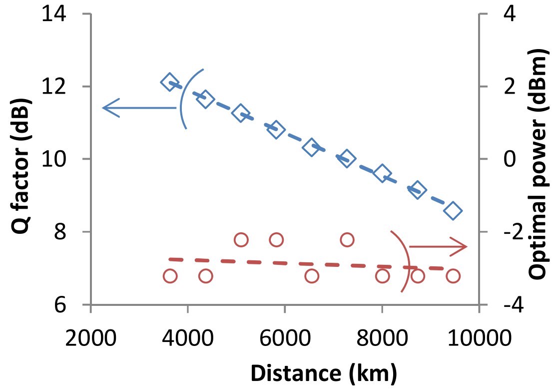

Figure 2: Q factor as a function of transmission distance measured at the optimal channel power.

Wavelength Selective Switch (WSS) to generate a comb of 52 channels with 50 GHz spacing. The receiver is a traditional coherent receiver with off line Digital Signal Processing (DSP) that performs: (i) chromatic dispersion compensation, (ii) I&Q imbalance compensation, (iii) retiming and resampling, (iv) polarization recovery and PMD compensation, (v) carrier frequency and phase recovery. A differential decoding is used to compensate for possible cycle slips.

From the Figure 2, we can extrapolate the maximum transmission distance the system can reach with respect to the design limit of the table 2 (9 dB). We obtain a reference system with a length of 8696km including 119 spans of 76 km and 11 equalizers working at the optimal power (-2.9 dBm).

Using this reference system, the results of equation (5') and (2') have been plotted in the Figure 3 and compared with the simulation ones. Equation (5') considering both the linear and non-linear transmission penalties gives a very good approximation of the optimal span length according to the system length whereas the equation (2') is clearly inefficient and can only be used with care for system very close to the system used as reference.

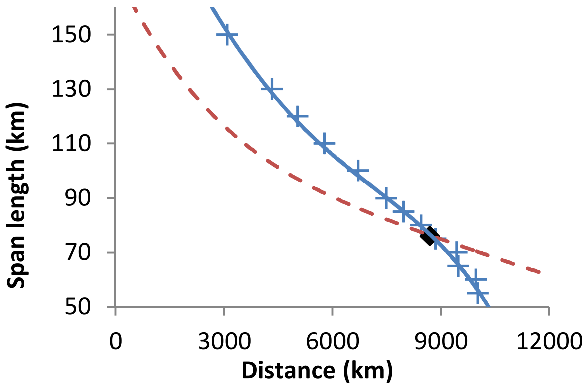

Figure 3: Maximum transmission distance as a function of span length. Crosses are the simulation results, the blue continuous line is the result of Equation (5) and the dashed red one comes from equation (2). The black diamond is the experimental reference system.

5. INTERPRETING THE VALUE OF THE COEFFICIENT β

It is interesting to understand the origin and the value of the coefficient β. In addition to the expression of amplification noise (used in section 2 above):

$$P_{ASE} = \eta_{ASE} \, . n \, . G \qquad (6)$$

we can also have a reasonably good approximation for P_{NL} from [2] with the assumption of a linear dependence in n as demonstrated in [4] for very long systems:

$$P_{NL} = a_{NL} \, . n \, . P^3 \qquad (7)$$

where a_{NL} is a parameter which does not depend on transmission distance or power.

The optimal power can easily be calculated by replacing P_{ASE} and P_{NL} into (3) and maximizing the SNR. It turns out to be:

$$P^{opt} = \sqrt[3]{\frac{\eta_{ASE} \, .G}{2 \, a_{NL}}} \qquad (8)$$

Therefore, the relationship between power P_0^{opt} and P^{opt} for the reference DLS and target DLS is:

$$\frac{P^{opt}}{P_0^{opt}} = \sqrt[3]{\frac{G}{G_0}} \qquad (9)$$

On the dB scale, this equation becomes:

$$P_{dB}^{opt} - P_{0\ dB}^{opt} = \frac{G_{dB} - G_{0\ dB}}{3} = \frac{\alpha}{3}(s - s_0) \qquad (10)$$

This provides the theoretical value $\beta = \frac{\alpha}{3}$. With $\alpha = 0.162$ dB/km, we obtain $\beta = 0.054$ that is consistent with the empirical value $\beta = 0.06$ found from simulations. However, the difference could be explained by the assumption that a_{NL} is independent of the span length, while in fact there is a small dependency.

6. APPROXIMATION ERROR OF THE SIMPLE FORMULAS VERSUS FULL SIMULATION

We have looked at the approximation error by comparing the estimations provided by the heuristic formulas to the values provided by the advanced simulation model.

Figure 4 shows the estimation error of the heuristic formula with respect to the advanced simulation model. The blue crosses represent the error when estimating the repeater number from equation (5) and (5') considering the repeaters are able to reach the optimum power. In that case, the maximum error never exceeds 10% corresponding to maximum 1 repeater difference between model and simulation whatever the DLS length.

As seen in Figure 1, the optimal power increase rapidly with the span length. However the repeater maximum output power is limited and sometimes doesn't achieve the optimal power per channel especially for short system with long span. This limitation is even stronger with the trend to improve the number of channels per fibre pair and can considerably reduce the maximum span length achievable. When the repeater maximum output power can not reach the optimal value, equations (5) and (5') lead to additional error when estimating the required repeater number (Figure 4). The error becomes large for short systems requiring large span length and therefore more power.

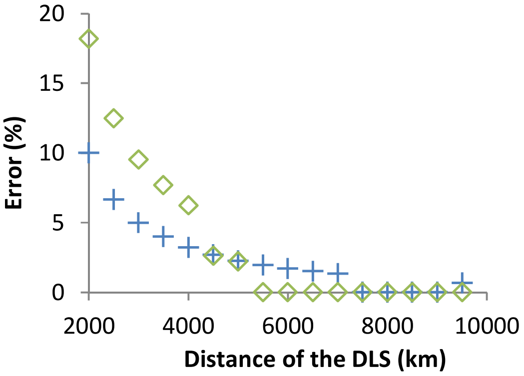

Figure 4: Absolute value of the percentage of error when using equation (5) and (5') to estimate the required number of repeaters according to DLS length, when systems are adjusted to the optimal power (blue crosses) and to maximum -2dBm per channel (green diamonds).

7. IMPACT OF THE LIMITED REPEATER OUTPUT POWER

To formulate the repeater output power limitation, we need to integrate the constraint when equalizing SNR for the reference and target systems. Considering SNR_0 is the SNR of the reference system adjusted at the optimal power, and using the property $P_{ASE} = 2P_{NL}$ at the optimal power [2], we have (as in section 5 above):

$$SNR_0 = \frac{P_0}{\frac{3}{2}\eta_{ASE} G_0 n_0} \qquad (11)$$

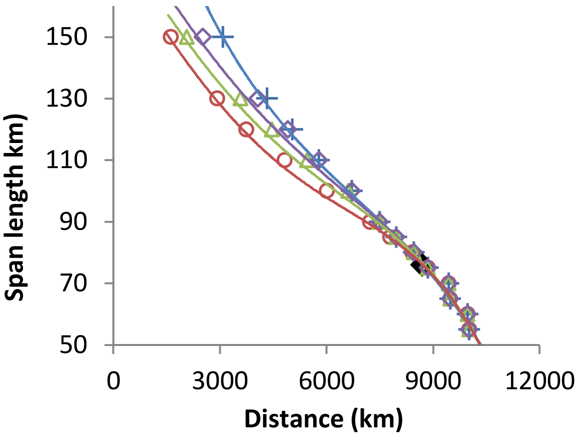

Figure 5: Maximum transmission distance as a function of span length and repeater maximum power. Crosses are simulation results with no power limitation while diamonds, triangles and circles have been constrained by a maximum power respectively, -1, -2 and -3dBm per channel. The continuous lines are the result of the set of equation (5') and (13').

We consider a new system which optimal power exceeds the maximum power that can be delivered by repeaters. We denote P_{max} the maximum power per channel. Noting from Equation (8) that:

$$a_{NL} = \eta_{ASE}\, G_0 / 2\, P_0^3 \quad ,$$

we can express the SNR of the new system as:

$$SNR = \frac{P_{max}}{\eta_{ASE}\, n\, G + \frac{\eta_{ASE}\, G_0}{2 P_0^3}\, n\, P_{max}^3} \qquad (12)$$

We then introduce the parameter $\gamma = \frac{P_{max}}{P_0}$, and equalize the SNR of equations (11) and (12) to obtain:

$$\frac{G}{G_0} = \frac{3}{2}\frac{n_0}{n}\gamma - \frac{\gamma^3}{2}$$

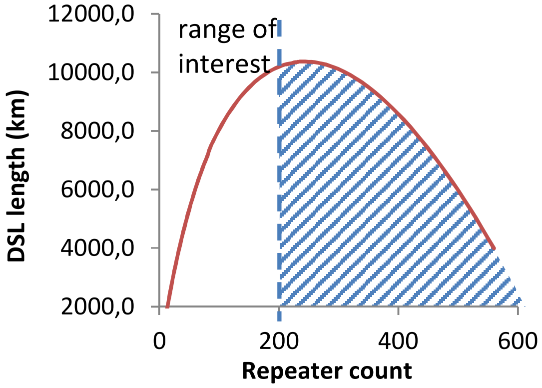

Figure 6: Overall length as a function of the number of repeaters in the case the repeaters can always reach the optimal power using β= 0.06.

The heuristic formulas for span length and overall length become:

$$s = s_0 + \frac{10}{\alpha}\log_{10}\left(\frac{3}{2}\frac{n_0}{n}\gamma - \frac{\gamma^3}{2}\right) \qquad (13)$$

$$l = \left[\frac{l_0}{n_0} + \frac{10}{\alpha}\log_{10}\left(\frac{3}{2}\frac{n_0}{n}\gamma - \frac{\gamma^3}{2}\right)\right] n \qquad (13')$$

To summarize, we can calculate an estimation of the longest span length usable for a given distance by using;

- Equations (5) and (5') when the maximum optical power of repeaters exceeds the required optimal power (10),
- Equations (13) and (13') when the repeater power cannot reach the optimal value.

In Figure 5 we compare the results obtained with this set of equations with simulations for different repeaters maximum output powers; -1, -2 and -3 dBm per channels respectively corresponding to 18, 17 and 16 dBm for a system equipped with 80 channels. As expected the power limitation of current amplifier can strongly reduce the maximum span length for short systems. This limitation will be even stronger with the use of next generation fibres with very larger effective area.

8. VARIATION OF THE DLS LENGTH WITH RESPECT TO THE NUMBER OF REPEATERS

Figure 6 shows the typical shape of the curve representing the overall length l as a function of the number of repeaters, using formula (5’) with $\beta = 0.06$.

The l function first increases up to the maximum achievable length of the DLS, and then decreases and returns to zero. Obviously, only the range of values where the curve increases is of practical interest. Moreover, the range of interest ends before reaching the maximum since in the vicinity of the maximum the expense of adding a repeater increases the overall length by a negligible amount.

9. CALCULATING THE NUMBER OF REPEATERS FOR A TARGET DLS LENGTH

Previous equations (5) and (13) allow to estimate the maximum span length for a given number of repeaters and (5’) and (13’) the maximum DLS length from the repeater number, both following the objective to have the same performance than the system used as a reference. However, the main objective is to evaluate the minimum repeater number for a target DLS length. To do this one needs to invert the function length l provided by equations (5’) and (13’).

A simple method (using a simple excel sheet) is just to tabulate the values of l to pick out the desired value of n.

An alternative method is to express (5’) under the following form:

$$n = \frac{l}{\frac{l_0}{n_0} - \frac{1}{\alpha-\beta} 10 \log_{10}\frac{n}{n_0}} = g(n) \qquad (14)$$

Thus for a given value of l , n is the solution of $n = g(n)$.

It can be calculated by iterating function $g(n)$ starting from a initial value of n. If we take $n = n_0$ as initial value, first iteration yields $n_1 = n_0 \; l/l_0$ which means we do not change the span, and the second iteration yields:

$$n_2 = l/\left(\frac{l_0}{n_0} - \frac{1}{\alpha-\beta} 10 \log_{10}\frac{l}{l_0}\right) \qquad (15)$$

This value can be accepted as an reasonable approximation for small changes of l . For higher changes, one should iterate up to convergence, which is typically reached within 10 iterations. The same approach can be taken with equation (13’).

10. DISCUSSION

Heuristic formula (5’) only requires the knowledge of the fibre attenuation, span length and overall length of the reference DLS while heuristic formula (13’) also requires the knowledge of the output power of the repeaters of the reference DLS as well as their maximum operating output power.

The underlying principle is to equate the SNR of a reference system and the SNR of target system. Therefore the hypothesis is that we keep the same power budget margin for the target system than for the reference system. Moreover, the strong hypotheses are the following:

- We are dealing with non compensated dispersion systems, as now deployed in conjunction with coherent detection technology.
- The reference system is tuned to operate at optimal power maximizing the SNR

The first assumption is easy to accept since several recent existing systems and all new systems are now designed that way.

The second assumption is more questionable. In fact, if the reference system is not designed at optimal power, it means that it has more margin available provided the repeater power can be tuned up. In that case using our simple approach results in transferring the additional margin to the target system.

One problem we addressed is the possible power output limitation of repeaters. This aspect comes into play when decreasing the length of the system. Having such repeater power limitation results into a higher number of repeaters. It will become more and more relevant as the systems transport more channels and use fibres with larger effective areas.

In this paper, we have also focused on the case of a long reference DLS, that one we could simulate with a laboratory circulation loop test bed. As a consequence, we studied mainly the case of achieving a target DLS by shortening the reference DLS. We have seen that the estimation error is low in that case (no more than one repeater count).

In fact, we have also looked at the case of extending the length of a short reference DLS, just by comparing the simulation model to formula (5’). When increasing the overall length, this estimation error gets significantly higher as the extrapolation ratio l/l_0 increases, but we have found that the results remains acceptable for the purpose of opportunity studies, up to a 50% extrapolation ratio.

11. CONCLUSION

The purpose of this paper is to help submarine cable planners, that are usually non specialists of submarine optical transmission, to study various architectural options of their projects. The heuristic formulas proposed here are designed for that purpose only, and not for precise line design. We have shown that such formulas can be used to estimate the repeater span of the longest DLS of a planned cable system from the knowledge just a few technical characteristics of an existing cable system already deployed in the field. We have compared this method to full line design simulation method, and we have verified that the error is low enough for the purpose of submarine cable opportunity studies.

12. REFERENCES

[1] J. Gaudette, P. Booi, E. Rivera Hartling, M. Andre and M. O’Sullivan, “Using coherent technology for simple, accurate performance budgeting”, Suboptic 2013, WE1A-2.

[2] F. Vacondio, O. Rival, C. Simonneau, E. Grellier, A. Bononi, L. Lorcy, J-C Antona, and S. Bigo, “On nonlinear distortions of highly dispersive optical coherent systems”, Optics Express, Vol. 20, No. 2, Jan. 2012.

[3] P. Poggiolini, “The GN Model of non-linear propagation in uncompensated coherent optical systems”, JLT, Vol. 30, N°. 24, dec. 2012.

[4] O. V. Sinkin, J.-X. Cai, D. G. Foursa, H. Zhang, A. N. Pilipetskii, G. Mohs, and Neal S. Bergano, “Scaling of Nonlinear Impairments in Dispersion-uncompensated Long-Haul Transmission”, OFC 2012, OTu1A.2.